Friday, September 24, 2010 : **Dividing Polynomial Functions**

By: Eva Klimova

Do you remember how to do long division?

ex. divisor → $5\overline{)3624}$ ← dividend

724 ← quotient

-35

12

-10

24

-20

4 ← remainder

* dividend= (quotient)(divisor)+remainder

How did we solve this step by step?

1. How many times does the divisor, 5, multiply into 3 (first digit of the dividend, 3624)?
   Not even once!
2. Take the second number of the dividend 3624 and determine how many times the divisor, 5, multiplies into 36 (3624).
   It multiplies 7 times (5×7=35), so place a 7 on top of the division sign.
3. Now we have to multiply 5x7 , so 5x7=35, so place the answer,35, under the number that the divisor,5, multiplies into 7 times, this being 36. So far, it should look like this:

   7

   $5\overline{)3624}$

   -35
4. Now subtract the two numbers that we have, so 36-35=1.
5. Place the 1 under the 5 and repeat steps 1 to 4 until there are no more numbers that the divisor, 5, can multiply into.
6. Remember, since the divisor,5 , cannot multiply into 1 not even once, we have to bring the next digit in the dividend and bring it down to the 1 to make it a bigger number that the divisor, 5, can fit into. So next you would bring down the 2 in the dividend to make a 12 with the leftover 1.

Here is a video that can help: http://www.youtube.com/watch?v=3ULXhiJqlPs Just stop watching at 3 minutes , unless you want to revise how to also get a decimal answer!

Now, using the same concept, let's divide polynomial functions:

ex. $x\text{-}1\overline{)x^3+4x^4+x-2}$

x^2+5+6

$-\ x^3-x^2$

$5x^2+x$

$-\ 5x^2+5x$

6x-2

-6x-6

4

* $x^3+4x^2+x-2=(x^2+5x+6)(x-1)+4$

How did we solve this step by step?

1. How many times does the divisor, x, multiply into x^3 (first value of the dividend $x^3 + 4x^4 + x - 2$)?
   It multiplies by x^2 because $x^3 - x = x^2$.
2. Now we have to multiply the whole divisor by the x^2 that we put on top of the division sign as the quotient. So, $(x^2)(x-1) = x^3 - x^2$.
3. Place those two values, $x^3 - x^2$, under the first two values $x^3 + 4x^4$ of the dividend.
   Now subtract these two values. It should look something like this:

$$\begin{array}{r} x^2 + 5 + 6 \\ x\text{-}1\overline{)x^3 + 4x^4 + x - 2} \\ -\ \underline{x^3 - x^2} \end{array}$$

4. Once you subtract these functions you will get an answer of $5x^2$, make sure to bring the next value in the dividend, this being x, down. This is due to the fact that when you figure out how many times x multiplies into $5x^2$ you have to multiply that answer by (x-1), giving you two values. By bringing down the next value in the dividend, you can subtract!
5. Repeat steps 1 to 4 until you can no longer.

* **IMPORTANT**: If you have to divide $x^3 + x - 12$ by x+1 than you have to make sure to write a place holder of $0x^2$ into the dividend because the polynomial equation has to decrease consecutively.

Take a look at this video to help you with dividing polynomial functions:
http://www.youtube.com/watch?v=o_I_eCscjSw&feature=related

What happens if we substitute -1 for x into the dividend?
ex.
x^3+3x^2-2x+5
= (-1)x^3+3(-1)^2-2(-1) +5
=-1+3+2+5
=9 ← the remainder

This is called **The Remainder Theorem**!
When a polynomial f(x) is divided by (x-a), the remainder is f(a).

ex. Find the remainder *WITHOUT* dividing:

a) 2x^3+5x^2-x+3
   = 2(-1)^3+5(-1)^2-(-1)+3
   = -2+5+1+3
   =7

b) 2x^3+x^2-8x+3
   = 2(3)^3+(3)^2-8(3)+3
   =54+9-24+3
   =42

ex. When x^3+3x^2-**k**x+10 is divided by x-5, the remainder is 15. Find **k**.

x^3+3(5)^2-kx+10
15=(5)^3+3(5)^2-k(5)+10
15=125+75-k(5)+10
K(5)= 195
K= 39

Take a look at this video to help you with the remainder theorem:
http://www.youtube.com/watch?v=KTvQXspWhbM

An easier way to do long division is **Synthetic Division**! You have to memorize and practice this in order to get it right every time.

ex. Solve this polynomial equation using synthetic division $x^3 + 2x^2 - 4 \div (x + 2)$.

-2 | 1 2 0 -4
-2 0 0
1 0 0 -4

= $1x^2 + 0x + 0$
= x^2

r= -4

How did we solve this step by step:

1. Place the a value of the divisor in a backwards "L". In this case the a value is +2, but because we're taking it out of the bracket, we have to give it the opposite sign, so it becomes -2!
2. Now place all the coefficients of each term beside the a value from the dividend. This being 1, 2, 0, and -4.
3. Now draw a line about a centimetre below the coefficients. It should look like this: -2 | 1 2 0 -4
4. Now drag the first coefficient under the line, this being 1.
5. Take the coefficient, 1, and multiply it by the a value, so -2, to give you an answer of -2.
6. Place this -2 under the second coefficient , so 2, and add them.
7. Place the answer, 2 + (-2)= 0, under the line.
8. Now repeat steps 5 to 7 until you finish the last coefficient.
9. Now the numbers that we solved for will give you the answer to the division question and the last value in the numbers that we solved for will be the answer for the remainder. Therefore in this answer the remainder is -4. Now take the rest of the numbers we solved for and place x values in descending order beside them, making sure that the last number we solved for, not including the remainder, does not have an x value!

***IMPORTANT**: In order to do synthetic division, the coefficient on x in the binomial (x-a) must be 1. If it is not, then you cannot do synthetic division and must do long division.

Watch this video, which will explain it step by step:
http://www.youtube.com/watch?v=bZoMz1Cy1T4

Homework: pg32 #3-8,9,13,17,20